Quantitative Methods for Decision-Making Under Uncertainty

Sankaran Mahadevan
Vanderbilt University, Nashville, TN

Email: sankaran.mahadevan@vanderbilt.edu
Website: www.reliability-studies.vanderbilt.edu
Reliability and Risk Engineering, Analysis, and Management

NSF-IGERT Graduate Program at Vanderbilt

Educational and Research Themes
- Multidisciplinary integration
- Large, complex systems
- Modeling and simulation
- Economic, legal, regulatory, and social perspectives

Cross-cutting methodologies
- Uncertainty quantification, propagation
- Risk quantification
- Decision-making under uncertainty

Participants
- 42 graduate students (34 Ph.D., 8 M.S.)
- 30 professors → Engineering, Math, Economics, Business, Psychology, Medicine
- Summer researchers (undergraduates, high school teachers)
Reliability and Risk Engineering, Analysis, and Management

Multiple Resources at Vanderbilt

- Structural/Mechanical Systems Reliability (S. Mahadevan)
- Institute for Space and Defense Electronics (R. Schrimpf)
- Institute for Software Integrated Systems (G. Karsai)
- Vanderbilt Center for Environmental Management Systems (VCEMS) (M. Abkowitz)
- VECTOR Transportation Research Center (M. Abkowitz)
- CRES P -- Multi-university Nuclear Waste Risk Assessment Consortium (D. Kosson)
- ACCRE High Performance Computing Network
- Business systems Risk Assessment and Management (B. Cooil)
- Multidisciplinary Doctoral Program in Reliability/Risk Engineering & Management
# Reliability and Risk Engineering, Analysis, and Management

## Industry & Government Support

### INDUSTRY
- Boeing
- Kellogg, Brown & Root
- Fedex
- General Motors
- Chrysler
- General Electric
- Pratt and Whitney
- Bell Helicopter
- Medtronic
- Union Pacific
- Xerox

### GOVERNMENT
- U. S. DOD (AFRL, Army, Navy, AFOSR)
- U. S. DOE (EM)
- U. S. DOT (FHWA)
- NASA (LaRC, MSFC, GRC, ARC, JPL)
- DOE Labs (SNL, LANL, INL, SRNL)
- Nuclear Regulatory Commission
- Federal Aviation Administration

### Laboratories
- Southwest Research Institute
- Transportation Technology Center

### Nature of support
- Summer internships
- Collaborative research projects
- Advisory committee membership
- Seminar speakers
- Student recruitment

---

Vanderbilt University

reliability-studies.vanderbilt.edu
Risk Analysis Issues

- System modeling
  - Physics-based behavior models → finite elements, bond graphs
  - Surrogate models (GP, PC, RBF, NN)
  - Fault trees, event trees, petri-nets
  - Bayes networks

- Risk analysis
  - Multi-level -- Material → component → subsystem → system
  - Risk variation over space and time
  - Multi-physics, multi-scale problems

- Data Uncertainty
  - Sparse data, interval data, measurement uncertainty
  - Expert opinion
  - Heterogeneous information

- Model Uncertainty
  - Model form, model parameters
  - Errors → some deterministic, some stochastic
Reliability and Risk Engineering, Analysis, and Management

- Materials durability, fatigue, fracture
- Systems health diagnosis and prognosis
- Decision-making under uncertainty
- Model uncertainty, calibration, validation
Rotorcraft Damage Tolerance (FAA)

Rotorcraft mast
• Two-diameter hollow cylinder
• Elliptical surface crack in fillet region
• Sub modeling technique
  Accuracy in stress intensity factor

Analyses
• Model calibration
  – Calibrate EIFS, model parameters
  – Estimate model errors in different stages of modeling
• Model validation
• Prediction uncertainty quantification
• Global sensitivity analysis
• Load monitoring and updating
Sources of Uncertainty

- Physical variability
  - Loading
  - Material Properties

- Data uncertainty
  - Sparseness of data used to quantify material properties
  - Output measurement uncertainty (final crack size, detection probability)

- Model uncertainty/errors
  - Analysis assumptions → LEFM, planar crack
  - Finite element discretization errors
  - Combination of multiple crack modes
  - Approximation due to surrogate model
  - Crack growth law → model form
  - Model parameters → crack growth model
    initial flaw size
Dynamic Bayes Network

\[ \theta \]

**Cycle \( i \)**
- \( a_i \)
- \( \Delta K \)
- \( C, m, n \)
- \( \varepsilon_{cg} \)
- \( \Delta K_{th}, \sigma_f \)

**Cycle \( i+1 \)**
- \( a_{i+1} \)
- \( \Delta K \)
- \( C, m, n \)
- \( \varepsilon_{cg} \)
- \( \Delta K_{th}, \sigma_f \)

\[ \varepsilon_{exp} \rightarrow a_N \rightarrow A \]
## Model Validation Metrics

<table>
<thead>
<tr>
<th>Model response</th>
<th>Observed values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$y$</td>
</tr>
</tbody>
</table>

**Null Hypothesis**

$H_0: y \in f(x)$

**Alternative Hypothesis**

$H_1: y \notin f(x)$

### Classical hypothesis testing

- **Hypotheses**
  - $H_0: E(y) = E(x)$  \hspace{5mm} $\text{Var}(y) = \text{Var}(x)$
  - $H_1: E(y) \neq E(x)$  \hspace{5mm} $\text{Var}(y) \neq \text{Var}(x)$

### Bayesian hypothesis testing

- From Bayes theorem:
  \[
  \frac{P(M_i | \text{observation})}{P(M_j | \text{observation})} = \frac{P(\text{observation} | M_i) P(M_i)}{P(\text{observation} | M_j) P(M_j)}
  \]

  - **Bayes Factor** $B$
  - “Confidence” $= B / (B + 1)$
Crack size prediction UQ

- Mean
- Lower Bound
- Upper Bound

Std. Dev.

Bayes Factor

Confidence
Sensitivity Analysis

• Local → Only one uncertainty is considered and all others are ‘frozen’ at the mean values

• Global → Analyze sensitivity of output over the entire domain of inputs rather than at mean values

• First order effects ($S$) & Total effects ($S_T$)

\[
S_i = \frac{V_{X_i}(E_{X_i}(Y | X_i))}{V(Y)}
\]

\[
S_{T_i} = \frac{E_{X_i}(V_X(Y | X_i))}{V(Y)} = 1 - \frac{V_{X_i}(E_X(Y | X_i))}{V(Y)}
\]
Cementitious barrier PA

Multi-physics analysis

Inputs
- Porosity
- Composition
- Modulus
- ...

Diffusion of Ions

Chemical Reactions

Damage Accumulation

Damage Progression

Durability prediction

Calibration of chemical reaction model parameters (equilibrium constants)
Extrapolation from Validation to Application

Bayes Network

Use for
- Calibration
- Validation
- Extrapolation

Extrapolation scenarios
- Nominal values to Extreme values
- Test conditions to Use conditions
- Validation variables to Decision variables
- Components to System

MCMC techniques
Gibbs sampling
UQ in system-level prediction

- **Level 0**: Foam, Material characterization
- **Level 1**: Joints, Component level
- **Level 2**: Sub-system level

Sources of uncertainty increase, system complexity increases, and amount of real data decreases.

Hardware data and photos courtesy of Sandia National Laboratories.

Vanderbilt University

reliability-studies.vanderbilt.edu
Bayes Network Implementation

Foam

\[ \theta^f \]

\[ X^f_1 \]

\[ \epsilon^f_1 \]

\[ X^f_2 \]

\[ \epsilon^f_2 \]

\[ Y^f_1 \]

\[ Y^f_2 \]

Joints

\[ \theta^j \]

\[ X^j_1 \]

\[ \epsilon^j_1 \]

\[ X^j_2 \]

\[ \epsilon^j_2 \]

\[ Y^j_1 \]

\[ Y^j_2 \]

\[ X_s \]

\[ Y = \text{Experimental data} \]

\[ X = \text{FEM prediction} \]

1 - Level 1

2 - Level 2

S - System

\[ J = \text{Joints} \]

\[ F = \text{Foam} \]

\[ \theta = \text{Calibration parameters} \]

\[ \epsilon = \text{Error terms} \]
Likelihood Approach to Data Uncertainty

- Likelihood function

\[ L(P) \propto \left( \prod_{i=1}^{n} \int_{a_i}^{b_i} f_X(x \mid P) dx \right) \left( \prod_{i=1}^{m} f_X(x_i \mid P) \right) \]

\( P \) – distribution parameters
\( m \) – point data size
\( n \) – interval data size

- Interval data
- Sparse point data

- Maximum Likelihood Estimate \( \rightarrow \) Maximize \( L(P) \)

- To account for uncertainty in \( P \) \( \rightarrow \)

\[ f(P) = \frac{L(P)}{\int L(P)} \]

- Two approaches
  - Family of distributions for \( X \) (for every sample of \( P \) \( \rightarrow \) probability distribution for \( X \))
  - Single distribution of \( X \)

\[ f(x) = \int f(x \mid P) f(P) dP \]

- Can use non-parametric distributions
Risk Management: System Health Monitoring

• System integration
  • Integrate reliability/risk methods with SHM
  • Integrate diagnosis with prognosis

• Rapid diagnosis and prognosis
  • Derive damage signatures
  • Qualitative isolation, then damage quantification

• Uncertainty Quantification
  • Quantify variability, uncertainty, errors
  • Estimate Confidence in diagnosis/prognosis
Decision-Making Under Uncertainty

- Various stages in life cycle → design, operations, maintenance
  - Multiple objectives, MCDA, decision trees, utility-based formulations
  - Multi-disciplinary systems
  - Optimization for reliability and robustness
  - Include both aleatory and epistemic uncertainties

- Dynamic, network systems
  - Critical facility protection – design of safeguards/detectors
  - Transportation networks, supply networks, emergency response systems

- System of systems
  - Multiple system linkages
  - Homeland security, military, commercial applications

Vanderbilt University

reliability-studies.vanderbilt.edu
Fire Satellite System

Analyses
- Multi-disciplinary uncertainty propagation
- Design optimization for reliability, robustness

Target latitude,
Target longitude
Target size
[Altitude]

Orbit period, eclipse period

Orbit
Orbit Period, Satellite velocity, Max slewing angle

Power
P_ACS
I_min, I_max

Attitude

[P_tot], [T_tot], [A_sat], [I_min], [I_max]
System of Systems Decision-Making Under Uncertainty

- **Decision making**
  - Risk-informed
  - Design
  - Operations
  - Utility theory

- **System Complexity**
  - Monolithic
  - Family of Systems
  - System of Systems

- **Optimization**
  - Non-linear
  - Stochastic
  - Static/Dynamic

- **Modeling**
  - Coefficient based
  - System dynamics
  - Agent based
  - Surrogate model

- **Human in the loop**
  - Fully rational
  - Bounded rational

- **Types of Systems**
  - Information bonded
  - Energy bonded
  - Hybrid

- **Uncertainty**
  - Aleatory
  - Epistemic

- **Risk**
  - Analysis
  - Management

Vanderbilt University
reliability-studies.vanderbilt.edu
Pandemic Influenza Risk Management

CIPDSS (LANL)

Scenario Description (Infectious Disease Outbreak)
- Initial Population
- City Data
- Outbreak Start
- Infectious Disease
- Stages
- Infectivity
- Normal Contact Rates
- Mortality

Disease Progression
- Contact Rates
- Infection Rates
- Immune Population
- Recovered Population
- Fatality Rates
- Infected Population

Response Operations
- Vaccine Supply
- Vaccine Distribution
- Treatment Facilities
- Treatment Staff
- Treatment Effectiveness
- Response Costs

Public Health
- Treatment Denial
- Hospital Beds
- Hospital Treatment
- Hospital Staff
- Hospital Costs

Emergency Services
- EMS Calls
- EMS Deployment
- EMS Costs

Government
- Disease Alert
- Response Policy
- Investments

Banking & Finance
- Business Operations
- Consumer Spending
- Revenue Loss

Information & Telecommunications
- Call Demand
- Call Attempts
- Lost Capacity
- Telecommunications Availability

Vanderbilt University
reliability-studies.vanderbilt.edu
Conclusion

Continuing opportunities for methods development

- System risk assessment
  - Risk variation with time and space
  - Dynamic, multi-physics, systems of systems
  - Computational effort
- Decision-making under uncertainty
  - Design, operations, maintenance, risk management
  - Data collection, Model development
  - Embedding flexibility
- Include data uncertainty
  - Sparse, noisy, qualitative, missing data, intervals, expert opinion
  - Multi-scale fusion of heterogeneous information
- Include model uncertainty
  - Validation, calibration, error estimation, extrapolation